51: Passive Solar Energy Book. Mazria. Rodale Press Em. Pa. USA 1979
0-87857-260-0

The Sun Charts

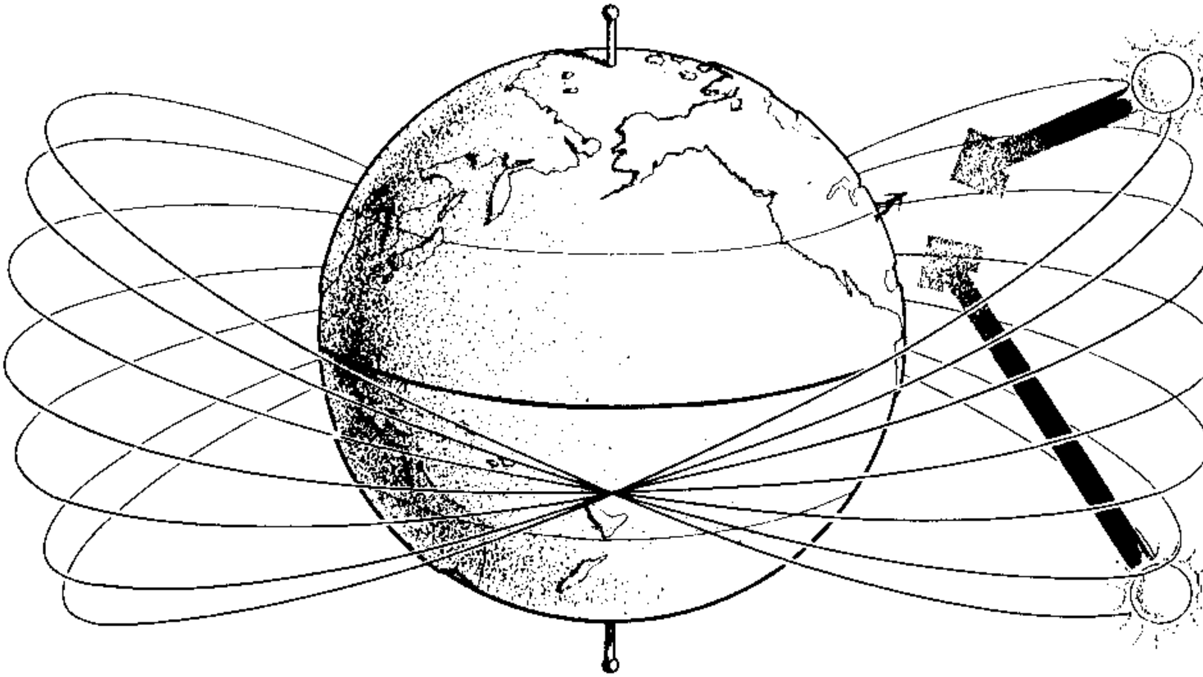
How the Sun Works

For our purposes, it is convenient to assume that the earth is stationary and the sun is in motion around the earth. Figure V-1 lists the angle (declination) of the sun above (+) or below (−) the equator, on the twentieth of each month, as seen from the earth. From the Northern Hemisphere, you can see that the sun lingers at its highest position in the sky for three months during the summer, then moves very quickly through fall towards winter, where it appears low in the sky for another three months.

In order to understand and be responsive to the effects of the sun on the location and design of places, it is necessary to know, at any given moment, the sun's position in the sky. This information is necessary in order to calculate solar heat gain, and to locate buildings, outdoor spaces, interior room arrangements, windows, shading devices, vegetation and solar collectors.

The Cylindrical Sun Chart

The Cylindrical Sun Chart, which is developed here, provides an easy-to-understand and convenient way to predict the sun's movement across the sky as seen from any point in the world between 28° and 56°NL. The chart is a vertical projection of the sun's path as seen from earth. It could be said, then, that the Sun Chart is an earth-based view of the sun's movement across the skydome.



The table below lists approximately how far above or below the equator the sun is on the twentieth day of each month.

20th of	Degrees
Jan.	−20
Feb.	−11
Mar.	0
Apr.	11
May	20
June	23
July	21
Aug.	13
Sept.	1
Oct.	−10
Nov.	−20
Dec.	−23

Fig. V–1: The sun as it appears from earth on the twentieth day of each month.

The following sequence is a description of how a sun chart is developed. It is included here to provide you with a visual understanding of the sun's movement across the skydome.

Two coordinates are needed to locate the position of the sun in the sky. They are called the *altitude* and *azimuth* (also called the bearing angle).

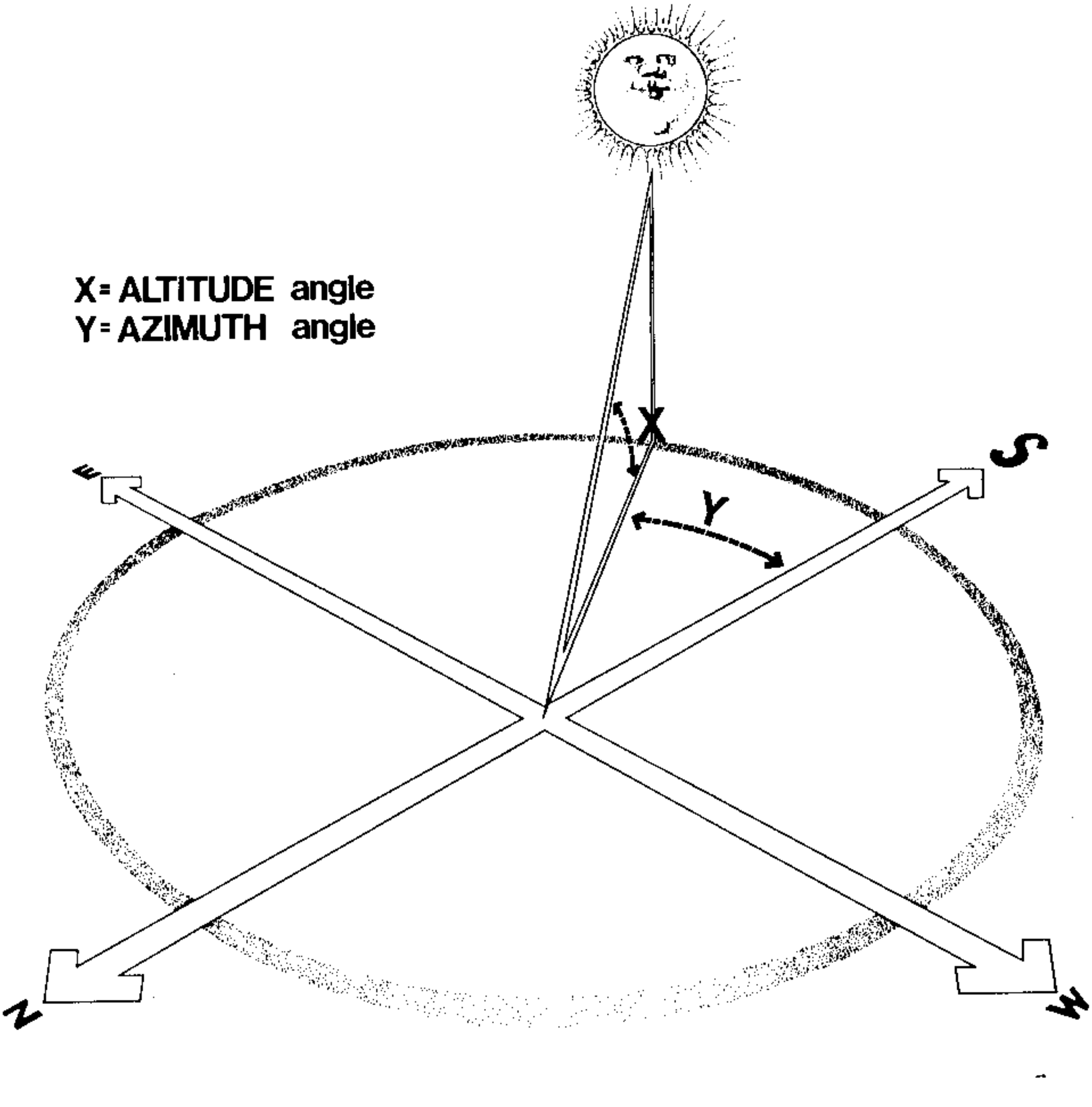


Fig. V–2: Altitude and azimuth angles.

Altitude

Solar altitude is the angle measured between the horizon and the position of the sun above the horizon. The horizontal lines on the chart represent altitude angles in 10° increments above the horizon.

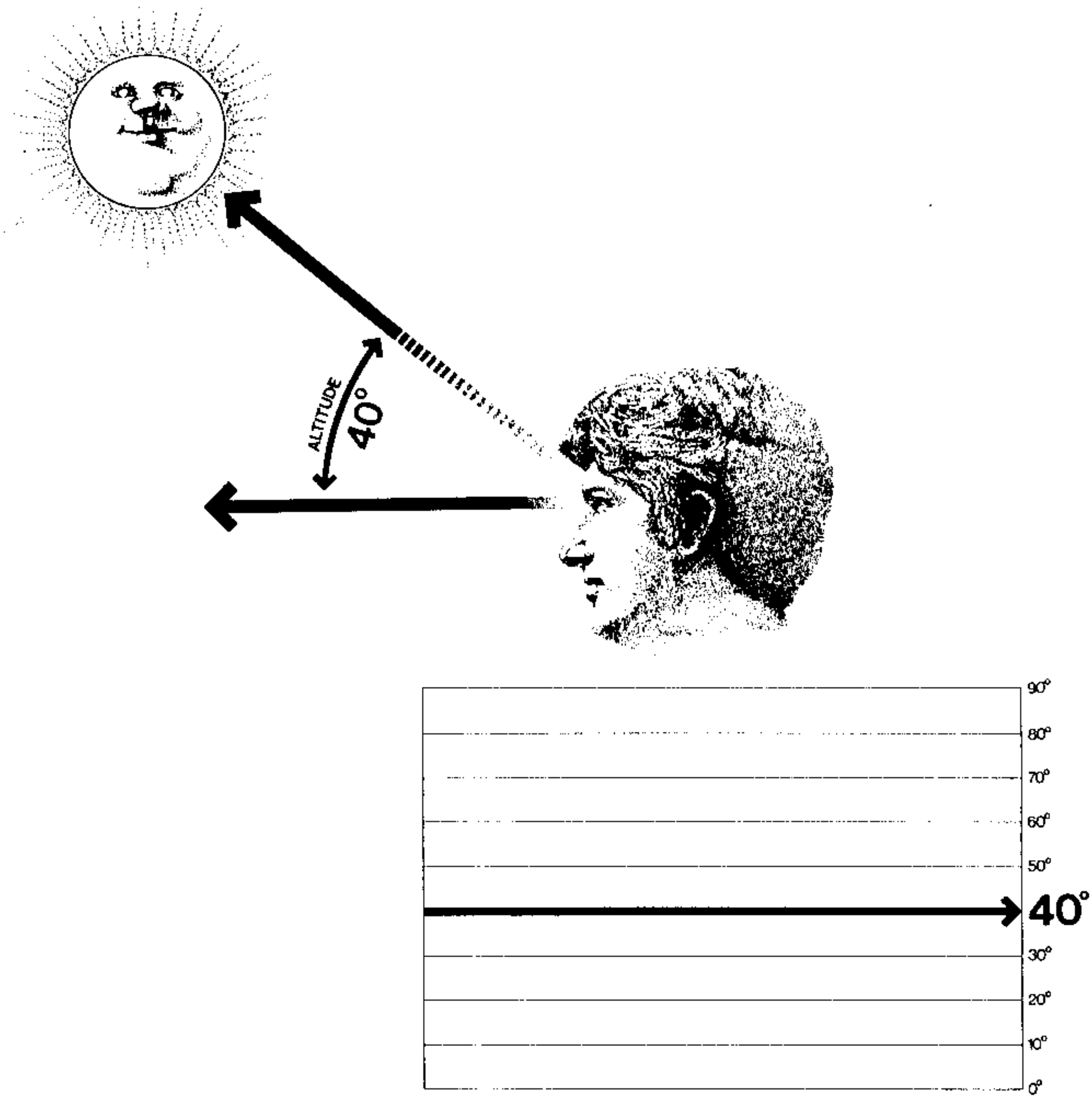


Fig. V–3: Altitude angle.

Azimuth (bearing angle)

Solar azimuth is the angle along the horizon of the position of the sun, measured to the east or west of true south.

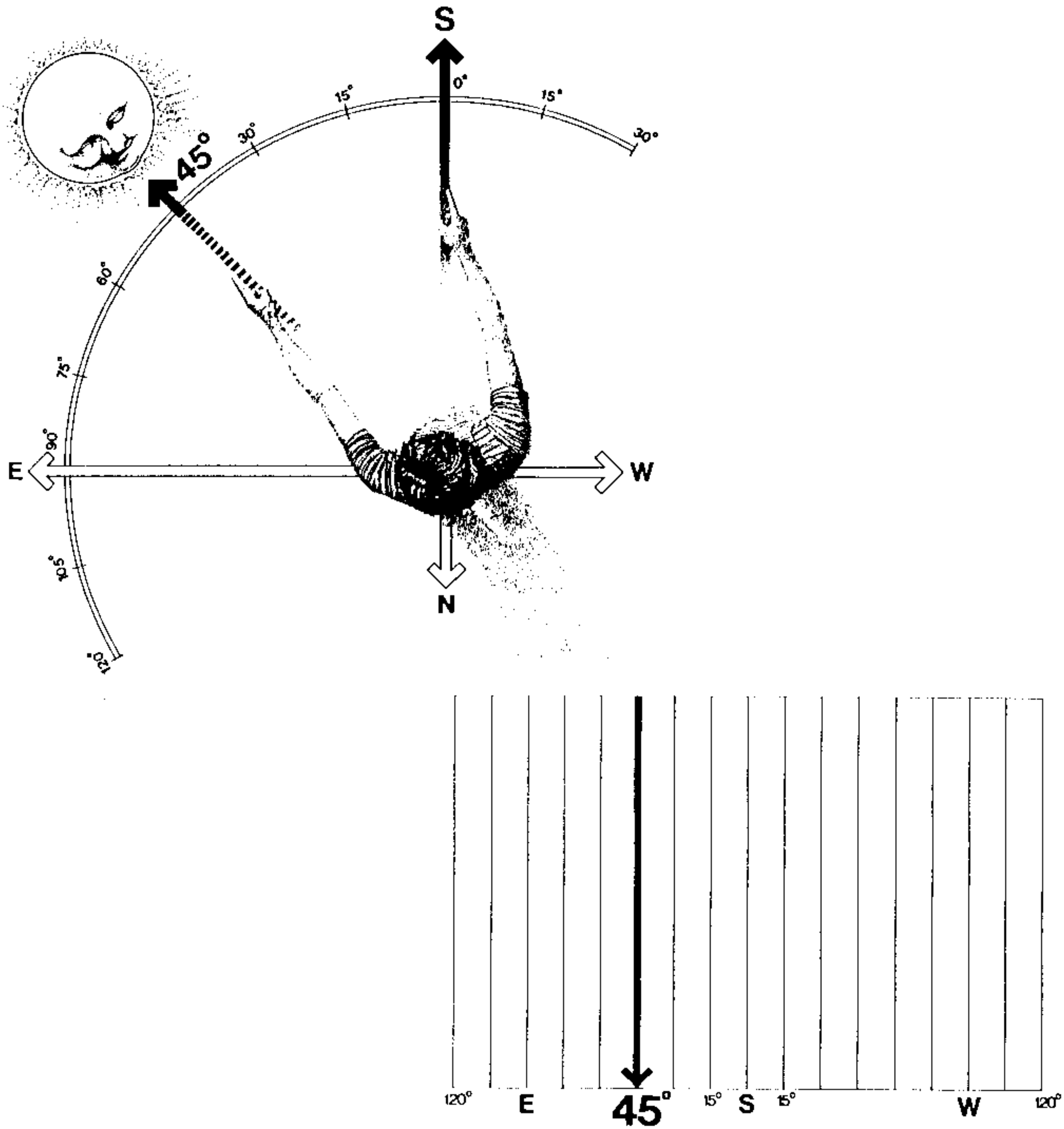


Fig. V–4: Azimuth angle.

Skydome (sky vault)

The skydome is the visible hemisphere of sky, above the horizon, in all directions. The grid on the chart represents the vertical and horizontal angles of the whole skydome. It is *as if* there were a clear dome around the observer, and then the chart were peeled off of this dome,* stretched out and laid flat.

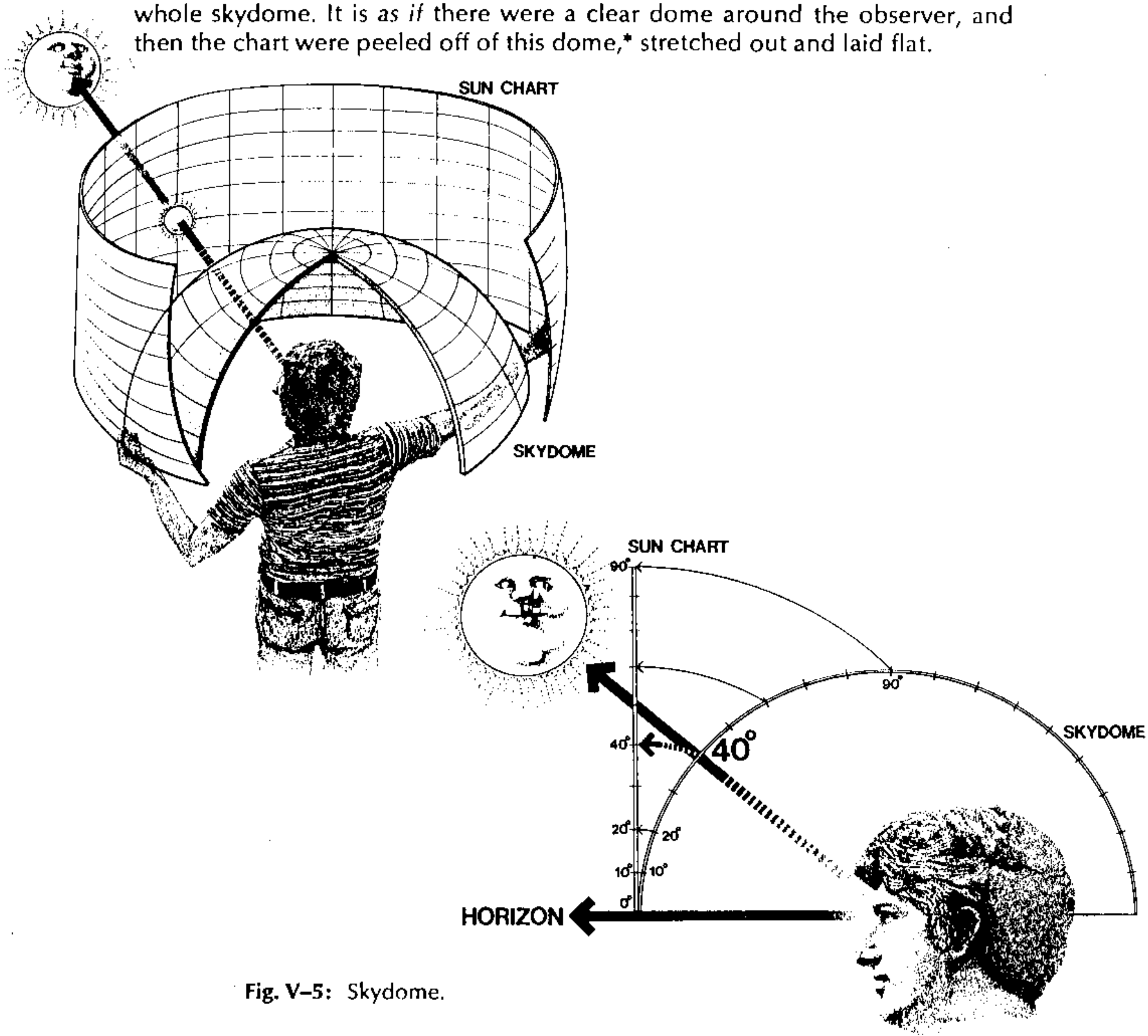
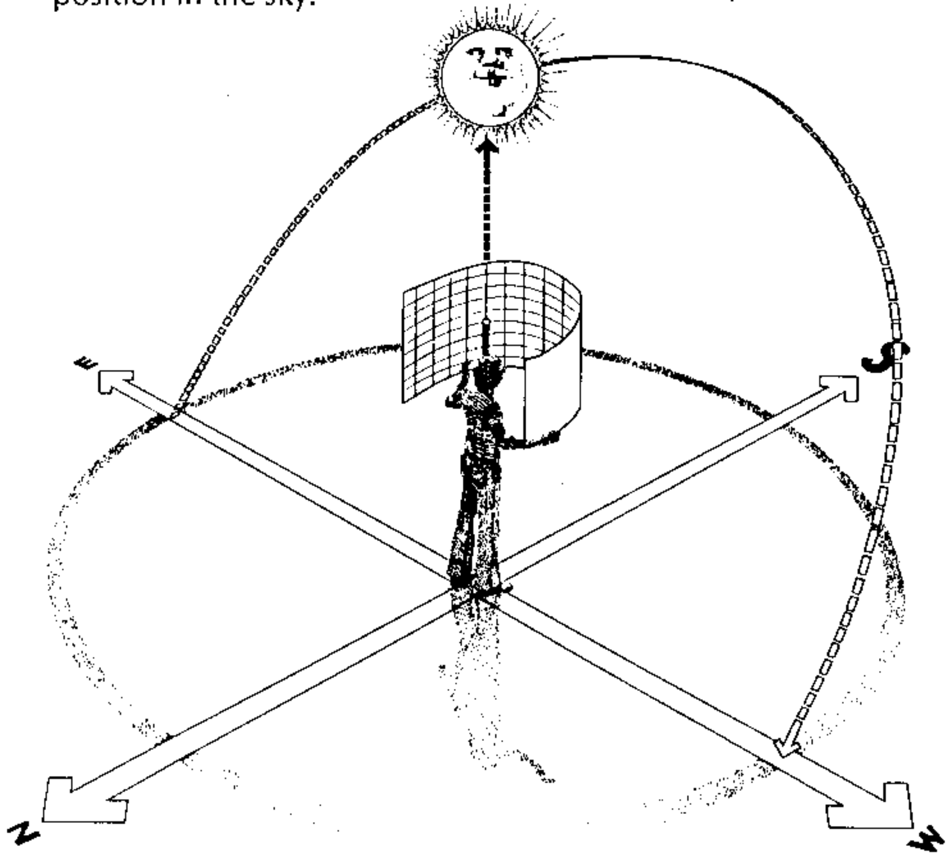


Fig. V–5: Skydome.

*In reality this is not possible. The intention of the illustration is to present you with a visual image of the skydome projected onto a flat sheet.

Sun's Position

Once the altitude and azimuth angles are known, the sun can be located at any position in the sky.



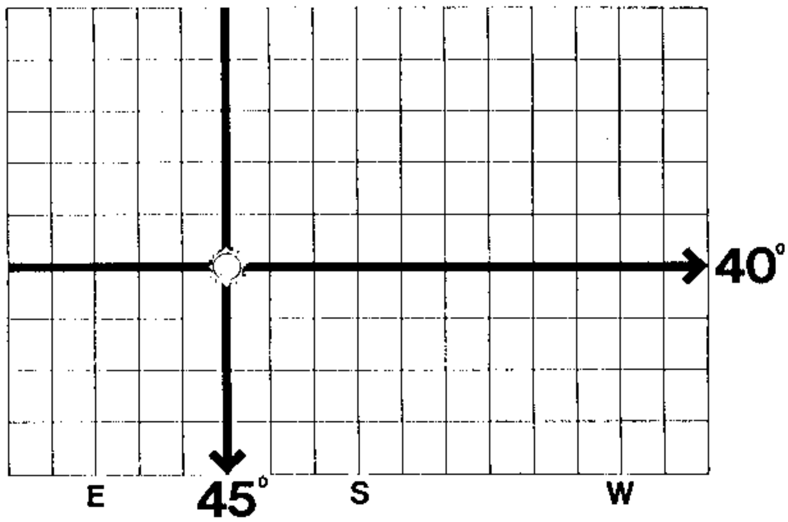
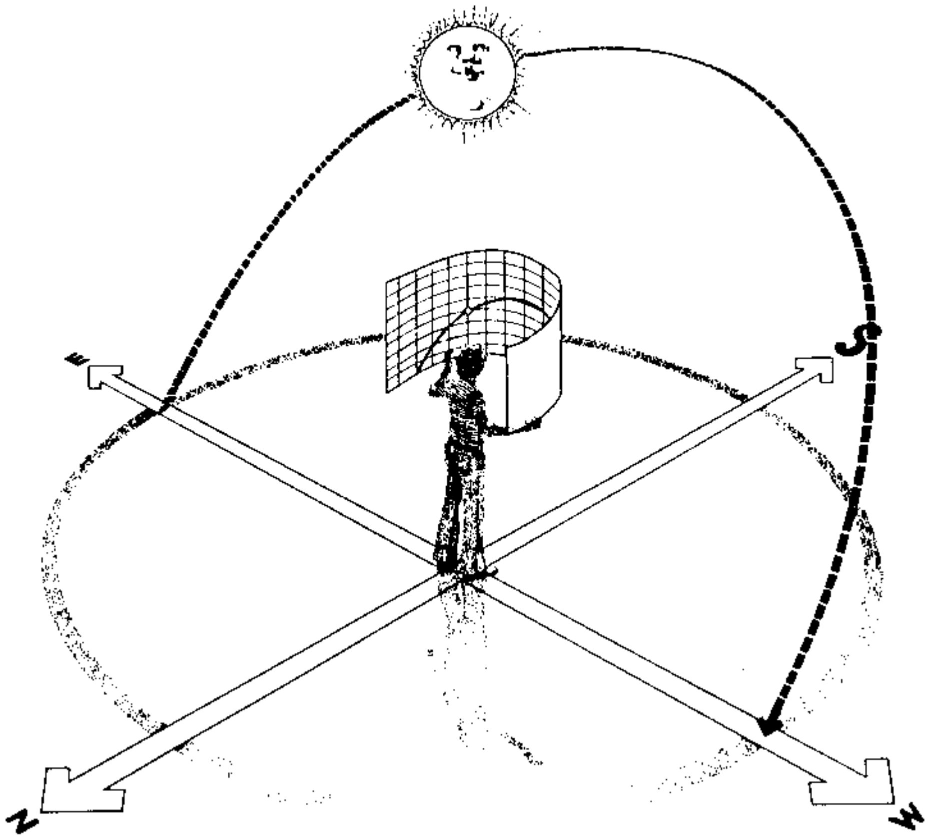


Fig. V–6: Sun's position.

Sun's Path

By connecting the points of the location of the sun, at different times throughout the day, the sun's path for that day can be drawn.



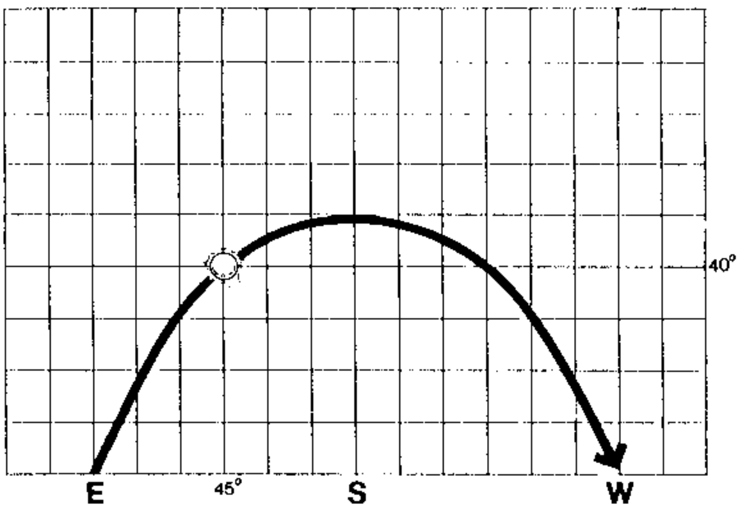
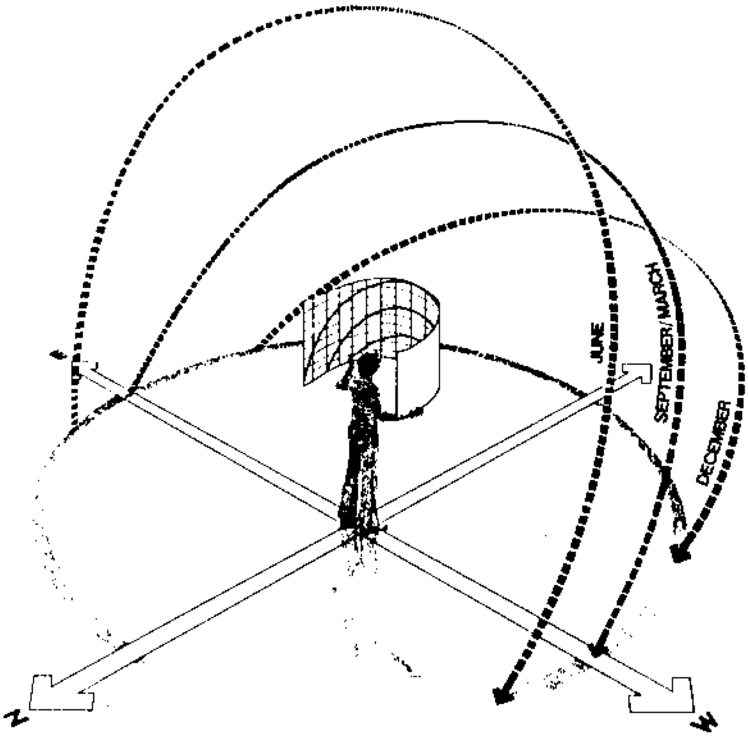


Fig. V–7: Sun's path.

Monthly Paths

Thus, we can plot the sun's path for any day of the year. The lines shown represent the sun's path for the twentieth day of each month. The sun's path is longest during the summer months when it reaches its highest altitude, rising and setting with the widest azimuth angle from true south. During the winter months the sun is much lower in the sky, rising and setting with the narrowest azimuth angles from true south.



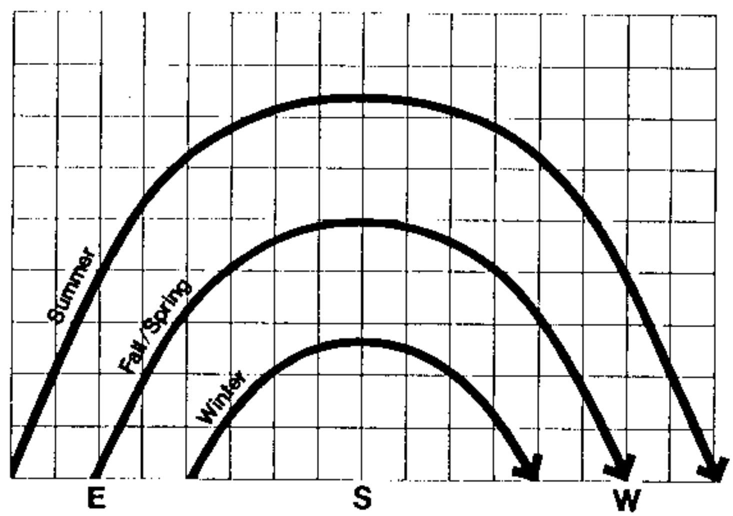
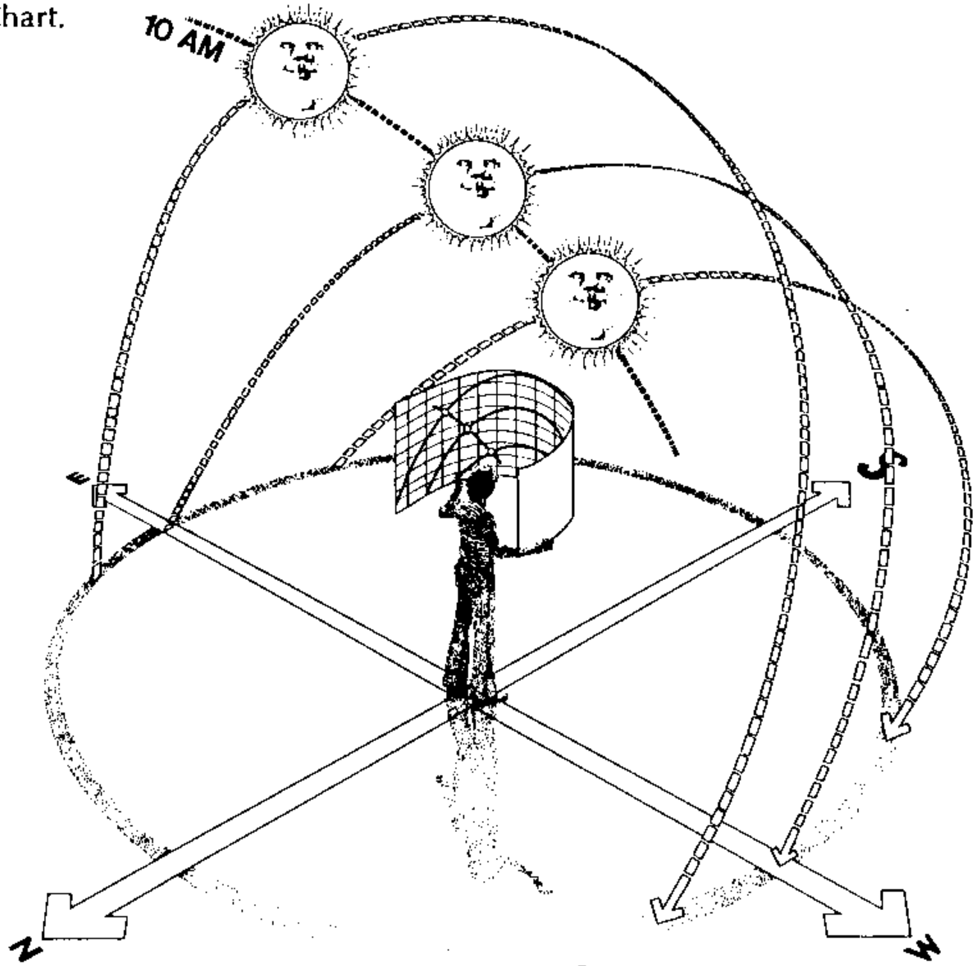


Fig. V–8: Monthly paths.

Times of Day

Finally, if we connect the times of day on each sun path we get a heavy dotted line which represents the hours of the day. This completes the Cylindrical Sun Chart.



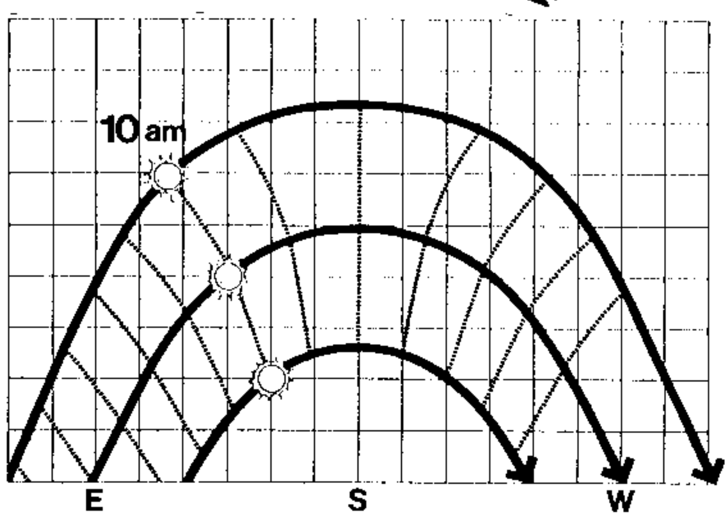


Fig. V–9: Times of day.

- **Solar Home Book. Anderson, Riordan. Cheshire Bks USA 1976. 0-917352-01-7.**

Solar Position

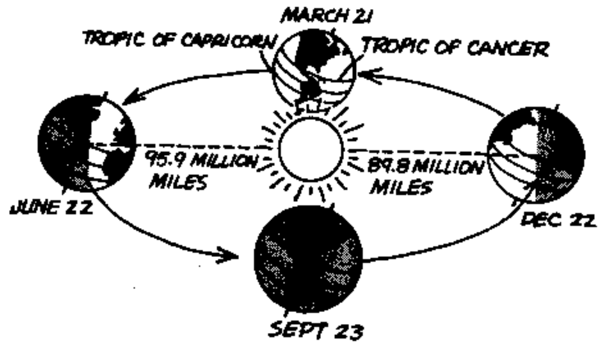
Most people have probably noticed that the sun is higher in the sky in snmmer than in winter. Some also realize that it rises south of due east in winter and north of due east in summer. Each day the sun travels in a circular path across the sky, reaching its highest point at noon. As winter proceeds into spring and summer, this circular path moves higher in the sky. The sun rises earlier in the day and sets later.

The actual position of the sun in the sky depends upon the latitude of the observer. At noon on March 21 and September 23, the vernal and autumnal *equinoxes*, the sun is directly overhead at the equator. At 40°N latitude, however, its angle above the horizon is 50° (= 90° − 40°). By noon on June 22, the summer *solstice* in the Northern Hemisphere, the sun is directly overhead at the Tropic of Cancer, 23½°N latitude. Its angle above the horizon at 40°N is 73½° (= 90° + 23½° − 40°), the highest it gets at this latitude. At noon on December 22, the sun is directly overhead at the Tropic of Capricorn, and its angle above the horizon at 40°N latitude is only 26½°.

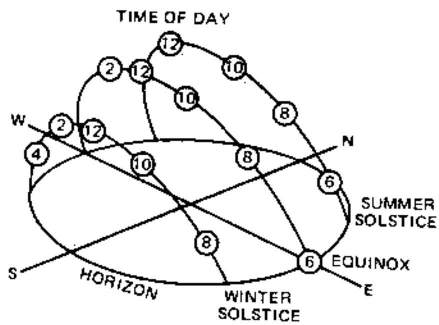
A more exact description of the sun's position is needed for most applications. In the language of trigonometry, this position is expressed by the values of two angles—the solar *altitude* and the solar *azimuth*. The solar altitude θ is measured up from the horizon to the sun, while the solar azimuth ϕ is the angular deviation from true south.

These angles need not be excessively mysterious—you can make a rough measurement of them with your own body. Stand facing the sun with one hand pointing toward it and the other pointing due south. Now drop the first hand so that it points to the horizon directly below the sun. The angle that your arm drops is the solar altitude θ and the angle between your arms in the final position is the solar azimuth ϕ. Much better accuracy can be obtained with better instruments, but the measurement process is essentially the same.

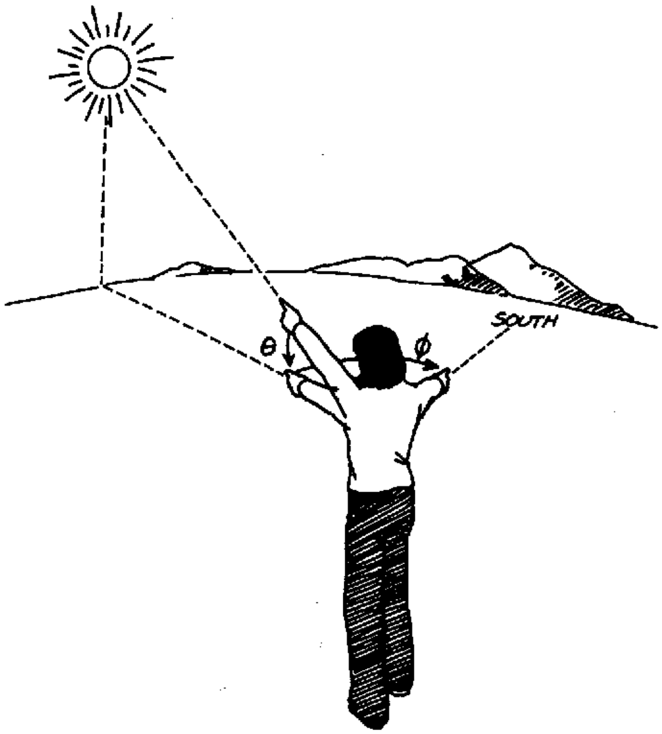
The solar altitude and azimuth can be calculated for any day, time, and latitude. For 40°N latitude (Philadelphia, for example),



The earth's elliptical path around the sun. The tilt of the earth's axis results in the seasons of the year.



The sun's daily path across the sky. The sun is higher in the sky in summer than in winter due to the tilt of the earth's axis.



Measuring the sun's position. The solar altitude θ is the angle between the sun and the horizon, and the azimuth ϕ is measured from true south.

the values of θ and ϕ are given at each hour for the 21st day of each month in the accompanying table. Note that ϕ is always zero at solar noon and that θ varies from 26.6° at noon on December 21 to 73.5° at noon on June 21. You can find similar data for latitudes 24°N, 32°N, 48°N, 56°N, and 64°N in the tables titled "Clear Day Insolation Data" in Appendix 1. This appendix also shows you how to calculate these angles directly for any day, time, and latitude.

SOLAR POSITIONS FOR 40°N LATITUDE

AM	PM	ANGLE	Jan 21	Feb 21	Mar 21	Apr 21	May 21	Jun 21	Jul 21	Aug 21	Sep 21	Oct 21	Nov 21	Dec 21
5	7	ALT θ					1.9	4.2	2.3					
		AZI ϕ					114.7	117.3	115.2					
6	6	ALT θ				7.4	12.7	14.8	13.1	7.9				
		AZI ϕ				98.9	105.6	108.4	106.1	99.5				
7	5	ALT θ		4.3	11.4	18.9	24.0	26.0	24.3	19.3	11.4	4.5		
		AZI ϕ		72.1	80.2	89.5	96.6	99.7	97.2	90.0	80.2	72.3		
8	4	ALT θ	8.1	14.8	22.5	30.3	35.4	37.4	35.8	30.7	22.5	15.0	8.2	5.5
		AZI ϕ	55.3	61.6	69.6	79.3	87.2	90.7	87.8	79.9	69.6	61.9	55.4	53.0
9	3	ALT θ	16.8	24.3	32.8	41.3	46.8	48.8	47.2	41.8	32.8	24.5	17.0	14.0
		AZI ϕ	44.0	49.7	57.3	67.2	76.0	80.2	76.7	67.9	57.3	49.8	44.1	41.9
10	2	ALT θ	23.8	32.1	41.6	51.2	57.5	59.8	57.9	51.7	41.6	32.4	24.0	20.7
		AZI ϕ	30.9	35.4	41.9	51.4	60.9	65.8	61.7	52.1	41.9	35.6	31.0	29.4
11	1	ALT θ	28.4	37.3	47.7	58.7	66.2	69.2	66.7	59.3	47.7	37.6	28.6	25.0
		AZI ϕ	16.0	18.6	22.6	29.2	37.1	41.9	37.9	29.7	22.6	18.7	16.1	15.2
12 noon		ALT θ	30.0	39.2	50.0	61.6	70.0	73.5	70.6	62.3	50.0	39.5	30.2	26.6
		AZI ϕ	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

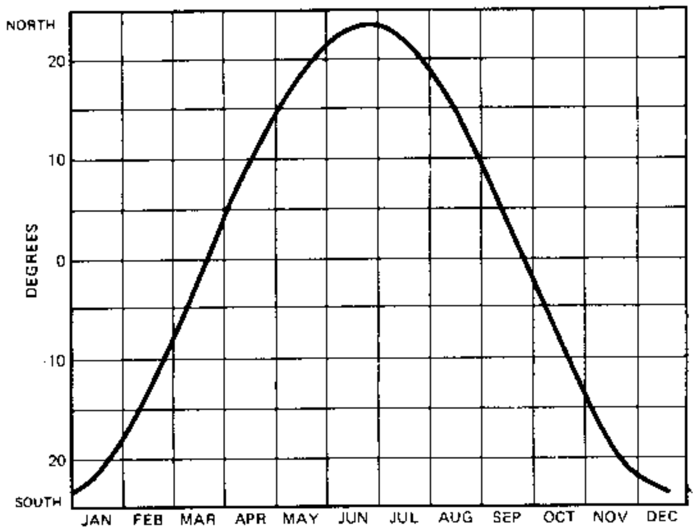
NOTES: Altitudes θ are measured from the horizon, and azimuths ϕ are measured from true south. Angles are given in degrees, and solar times are used.

SOURCE: Koolshade Corporation.

Appendix 1.1: SOLAR ANGLES

The sun's position in the sky is described by two angular measurements, the solar altitude θ and the solar azimuth ϕ. As explained in Chapter 3, the solar altitude is the angle of the sun above the horizon. The azimuth is its angular deviation from true south.

The exact calculation of θ and ϕ depends upon three variables: the latitude L, the declination δ, and the hour angle H. Latitude is the angular distance of the observer north or south of the equator—it can be read from any good map. Declination is a measure of how far north or south of the equator the sun has moved. At the summer solstice $\delta = +23\frac{1}{2}°$, while at the winter solstice $\delta = -23\frac{1}{2}°$ in the Northern Hemisphere; at both equinoxes, $\delta = 0°$. This qnantity varies from month to month and can be read directly from the graph below.



DECLINATION OF THE SUN

The hour angle H depends on Local Solar Time, which is the time that would be read from a sundial oriented south. Solar Time is measured from solar noon—the moment when the sun is highest in the sky. At different times of the year, the lengths of solar days (measnred from solar noon to solar noon) are slightly different from days measured by a clock running at a uniform rate. Local Solar Time is calculated taking this difference into account. There is also a correction if the observer is not on the standard time meridian for his time zone.

To correct local standard time (read from an accurate clock) to Local Solar Time, three steps are necessary:

1) If daylight savings time is in effect, subtract one hour.
2) Determine the longitude of the locality and the longitude of the standard time meridian (75° for Eastern ST, 90° for Central ST, 105° for Mountain ST, 120° for Pacific ST, 135° for Yukon ST, 150° for Alaska-Hawaii ST). Multiply the difference in longitudes by 4 minutes/degree. If the locality is east of the standard meridian, add the correction minutes; if it's west, subtract them.
3) Add the equation of time (from the next graph) for the date in question. The result is the Local Solar Time.

Once you know the Local Solar Time, you can obtain the hour angle H from:

$$H = 0.25 \times \text{(number of minutes from solar noon)}.$$

From the latitude L, declination δ and hour angle H, the solar altitude θ and azimuth ϕ follow after a little trigonometry:

$$\sin\theta = \cos L \cos\delta \cos H + \sin L \sin\delta\,;$$
$$\sin\phi = \cos\delta \sin H/\cos\theta\ .$$

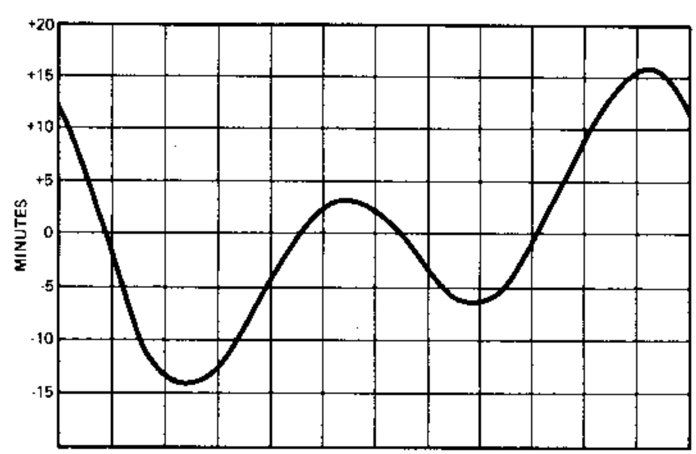
Example: Determine the altitude and azimuth of the sun in Abilene, Texas on December 1, when it is 1:30 p.m. (CST). First we need to calculate the Local Solar Time. It is not daylight savings time, so no correction for that is needed. Looking at a map we see that Abilene is on the 100°W meridian, or 10 degrees west of the standard meridian—90°W. We snbtract the $4 \times 10 = 40$ minutes from local time; 1:30 — 0:40 = 12:50 p.m. From the equation of time for December 1, we mnst *add* about 11 minutes. 12:50 + 0:11 = 1:01 Local Solar Time, or 61 minutes past solar noon. Consequently, the hour angle is $H = 0.25 \times 61$ or about 15°. The latitude of Abilene is read from the same map: L = 32°, and the declination for December 1 is $\delta = -22°$. We have come thus far with maps, graphs, and the back of an envelope, but now we need a pocket calculator or a table of trigonometric functions:

$$\begin{aligned}\sin\theta &= \cos(32°)\cos(-22°)\cos(15°) + \sin(32°)\sin(-22°)\\ &= 0.85 \times 0.93 \times 0.97 + 0.53 \times (-0.37)\\ &= 0.76 - 0.20 = 0.56\end{aligned}$$

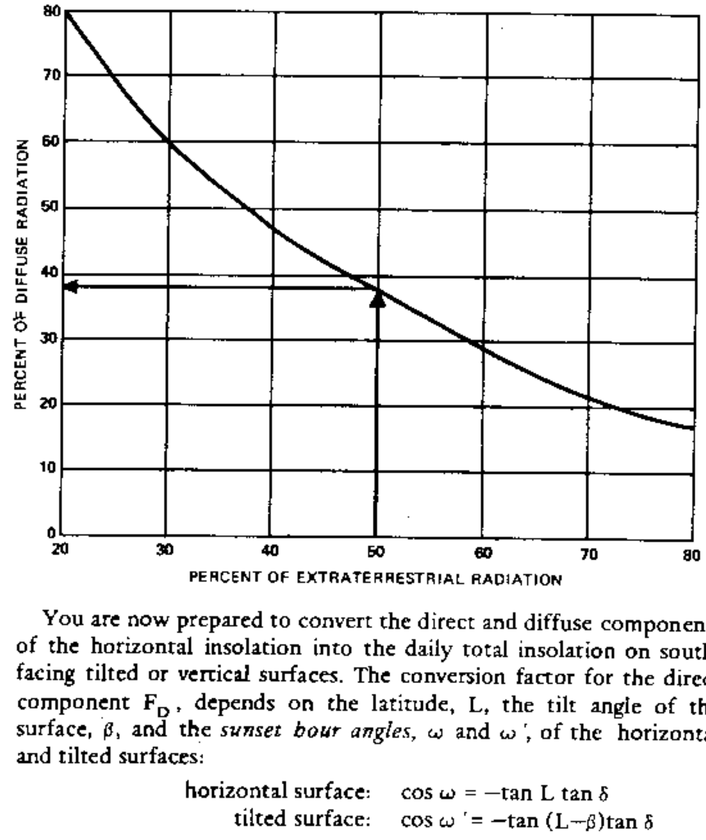
Then $\theta = \arcsin(0.56) = 34.12°$ above the horizon. Similarly:

$$\begin{aligned}\sin\theta &= \cos(-22°)\sin(15°)/\cos(34.12°)\\ &= (0.93 \times 0.26)/0.83 = 0.29\ .\end{aligned}$$

Then $\phi = \arcsin(0.29) = 16.85°$ west of true south. At 1:30 p.m. on December 1 in Abilene Texas, the solar altitude is 34.12° and the azimuth is 16.85° west.



The total solar radiation is the sum of direct, diffuse, and reflected radiation. At present, a statistical approach is the only reliable method of separating out the diffuse component of horizontal insolation. The full detail of this method is contained in an article by Liu and Jordan; we only summarize their results here. First we ascertain the ratio of the daily insolation on a horizontal surface (measured at a particular weather station) to the extraterrestrial radiation on another horizontal surface (outside the atmosphere). This ratio (usually called the *percent of Extraterrestrial radiation*, or % ETR) can be determined from the National Weather Records Center; it is also given in the article by Liu and Jordan. With a knowledge of the % ETR, you can use the accompanying graph to determine the percentage of diffuse radiation of a horizontal surface. For example, 50% ETR corresponds to 38% diffuse radiation and 62% direct radiation.



You are now prepared to convert the direct and diffuse components of the horizontal insolation into the daily total insolation on south-facing tilted or vertical surfaces. The conversion factor for the direct component F_D, depends on the latitude, L, the tilt angle of the surface, β, and the *sunset hour angles*, ω and ω', of the horizontal and tilted surfaces:

$$\text{horizontal surface:} \quad \cos\omega = -\tan L \tan\delta$$
$$\text{tilted surface:} \quad \cos\omega' = -\tan(L-\beta)\tan\delta$$

where the declination δ is found from the graph on page 252 and $\beta = 90°$ applies to vertical surfaces. Depending on the value of these two angles ω and ω', the calculation of F_D is slightly different. If ω is less than ω', then

$$F_D = \frac{\cos(L-\beta)}{\cos L} \times \frac{\sin\omega - \omega\cos\omega'}{\sin\omega - \omega\cos\omega}$$

If ω' is smaller than ω, then

$$F_D = \frac{\cos(L-\beta)}{\cos L} \times \frac{\sin\omega' - \omega'\cos\omega'}{\sin\omega - \omega\cos\omega}$$

The direct component of the radiation on a tilted or vertical surface is $I_D' = F_D \times I_D$, where I_D is the direct horizontal insolation.

The treatment of diffuse and reflected radiation is a bit different. The diffuse radiation is assumed to come uniformly from all corners of the sky, so one need only determine the fraction of the sky exposed to a tilted surface and reduce the horizontal diffuse radiation accordingly. The diffuse radiation on a surface tilted at an angle β is

$$I_d' = \frac{1+\cos\beta}{2} \times I_d$$

where I_d is the daily horizontal diffuse radiation. The reflected radiation on a tilted surface is

$$I_r' = \rho \times \frac{1-\cos\beta}{2} \times (I_D + I_d)$$

where ρ is the reflectance of the horizontal surface.

SOURCE: Liu, B.Y.H. and R. C. Jordan, "Availability of Solar Energy for Flat-Plate Solar Heat Collectors." in *Low Temperature Engineering Applications of Solar Energy*, edited by Richard C. Jordan, New York: ASHRAE, 1967.

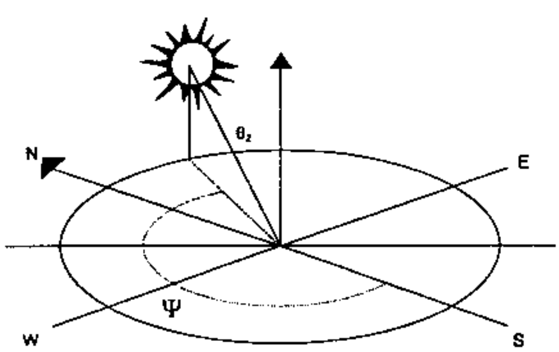
RAPS. University of Cape Town. South Africa 1992. 0-7992-1435-3.

EQUATIONS FOR CALCULATING THE POSITION OF THE SUN

Nomenclature:

ϕ	latitude
n	Julian day number
ω	hour angle
δ	solar declination angle
θ_z	solar zenith angle
ψ	solar azimuth

The apparent position of the sun in the sky can be defined by two angles: the solar zenith angle θ_z, and the azimuth angle ψ.



y is zero at solar noon, positive in the mornings and negative in the afternoons.

The sun's position relative to the earth depends on the time of day, the time of year, and the latitude.

The latitude ϕ is negative for the southern hemisphere. The time of year n is denoted by the Julian day number (1 to 365).

The hour angle of the sun ω is defined in terms of the number of hours from solar noon. ω changes by 15° every hour. It is zero at solar noon, positive in the mornings and negative in the afternoons. For a given time of day (solar time),

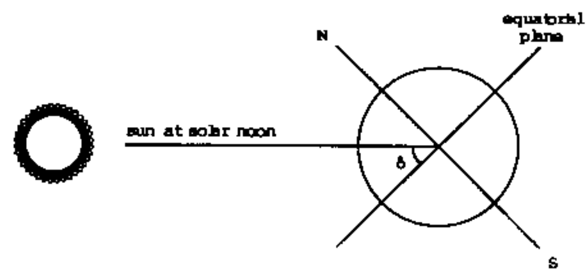
$$\omega = (noon - time) * 15^\circ$$

Solar declination angle

The declination angle of the sun δ is the angle between the sun at solar noon and the equatorial plane. This varies from day to day. It is zero at the equinoxes, 23.45° at the June solstice and -23.45° at the December solstice.

δ can be calculated as follows:

$$\delta = 23.45^\circ \times sin(360^\circ \times \frac{n + 284}{365})$$



Solar zenith angle

The solar zenith angle can now be calculated:

$$\cos \theta_z = \sin \delta \sin \varphi + \cos \delta \cos \varphi \cos \omega$$

$$for\ cos\ \omega > 0$$

Azimuth angle

The azimuth angle ψ is provided by

$$\cos \psi = \frac{\cos \theta_z \sin \varphi - \sin \delta}{\sin \theta_z \cos \varphi}$$

Appendix: Calculation of solar irradiation on inclined surfaces

NOMENCLATURE

Solar terms

D_h = diffuse radiation on the horizontal (W/m^2)

D_t = diffuse radiation on tilted plane (W/m^2)

G_h = global Radiation on the horizontal (W/m^2)

I = direct radiation on plane normal to sun's rays (W/m^2)

I_o = normal extraterrestrial radiation ($I_o = 1367\ W/m^2$)

I_θ = direct radiation on plane with solar incidence = θ

Ref = reflected radiation on tilted plane (W/m^2)

m = relative air mass

Solar angles

A = solar azimuth

E = solar elevation

z = solar zenith

δ = declination angle

θ_t = solar incidence angle on tilted plane

θ_h = solar incidence angle on horizontal

ω = hour angle

Site dependent terms

A_t = azimuth of tilted plane

n = day number

β = tilt angle of plane from horizontal

ϕ = latitude ($\phi > 0$ in southern hemisphere)

σ = albedo of surrounding ground cover

Perez model variables

F1, F2, L = sky brightness parameters

Fij = coefficients describing sky brightness parameters

a′, b′ = solid angles occupied by circumsolar zone and horizon band weighted by their average incidence on the slope

c′, d′ = the equivalent of a and b for the horizontal

ϵ = describes relative importance of direct radiation at the earth's surface

d = normalised horizontal diffuse radiation

α = half angle of circumsolar zone ($\alpha = 25^o$)

ϕh = term used in the calculation of c′

ϕt = term used in the calculation of a′

Xh = term used in the calculation of c′

Xt = term used in the calculation of a′

The radiation received by an inclined surface is different to that received on the horizontal for three reasons. Firstly, the direct component is altered because of the change in surface area projected onto the plane normal to the sun's rays. Secondly an inclined surface will receive radiation reflected from the surrounding ground cover. Thirdly, the diffuse component changes as the fraction of the sky dome visible to the tilted surface is reduced. It is current practice to treat these three components independently.

The direct component

The treatment of the direct component is relatively straightforward and error free for flat surfaces.

If I is the intensity of the solar radiation falling on a plane normal to the sun's rays, then the direct radiation, I_θ, falling on a plane where the solar incidence angle is θ, is given by

$$I_\theta = I \cos\theta$$

If θ_t is the solar incidence angle on the inclined plane and θ_h is the solar incidence angle on the horizontal, then

$$I_{\theta t} = I \cos\theta_t$$

$$\text{and } I_{\theta h} = I \cos\theta_h$$

so

$$I_{\theta t} = (\cos\theta_t / \cos\theta_h)\, I_{\theta h}$$

The ratio $\cos\theta_t / \cos\theta_h$ may be written:

$$\frac{\cos\theta_t}{\cos\theta_h} = \frac{\sin\delta \sin(\phi-\beta) + \cos\delta\cos(\phi-\beta)\cos\omega}{\sin\delta\sin\phi + \cos\delta\cos\phi\cos\omega}$$

where

δ = declination angle
ϕ = latitude
β = tilt of plane from horizontal
ω = hour angle

It is possible to repeat this calculation with data for each hour during the day and sum the results. The final figure represents the total direct radiation received during one day by the inclined plane.

The reflected component

The model employed to estimate ground reflected radiation assumes that the surrounding ground cover reflects radiation isotropically. This assumption is valid when global radiation is composed primarily of diffuse radiation and/or where the ground cover is a perfectly diffuse reflector. Although there do exist anisotropic models, these should only be used under specific conditions, for example where a surface exhibits strong directional reflectance or where local obstructions to the horizon occur.

For the isotropic model, the reflected component is given by:

$$Ref = 0.5\,\sigma\,(1 - \cos\beta)\,G_h$$

where

Ref	=	reflected component
σ	=	ground albedo
β	=	tilt of plane from horizontal
G_h	=	global radiation on the horizontal

The diffuse component

The available models

Estimation of the diffuse component of radiation received by an inclined plane is considered the largest potential source of error. There are a number of different models which may be used to predict the diffuse component.

Firstly, the isotropic model assumes that the intensity of the sky diffuse radiation is uniform over the entire sky dome. Research has shown that the assumption of isotropy of the sky provides a good fit to empirical data at low intensity conditions found during overcast skies; however the model underestimates the amount of solar radiation falling on tilted surfaces at higher solar intensities and in clear or partly clear sky situations where anisotropic conditions of circumsolar and horizon brightening are prevalent.

Various anistropic models have been developed to improve accuracy (Temps & Coulson, 1977; Klucher, 1979; Hay & McKay, 1985; and Perez et al, 1986). Comparisons of the performance of these models on two test data sets are presented in Perez (1987). The new version of the Perez model has been shown to perform more accurately than other models for a large number of locations. For this reason the new enhanced form of the Perez model has been selected to generate the diffuse radiation component.

Although Southern African sky conditions, being mostly clear and bright, resemble the conditions under which the Perez model performs well, it has not yet been validated locally.

Description of Enhanced Perez Model

The model is composed of two distinct elements:

i. a geometric representation of the sky dome

ii. a parametric representation of the insolation conditions.

The geometric framework

As shown in Figure 1, the sky hemisphere is divided into three zones: the horizon band, the circumsolar region and the rest of the sky. The diffuse radiation is assumed to be constant within each zone. Such a configuration helps to account for the two main types of anisotropy in the atmosphere: circumsolar and horizonal brightening. A 25° half angle for the circumsolar region was found by Perez to provide the best overall performance.

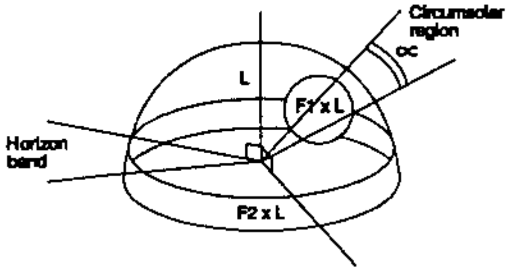


Figure 1: Geometric Representation of Sky Dome

If the diffuse radiances originating from the main portion of the dome, the circumsolar zone and the horizon band are L, F1 * L and F2 * L respectively, then the resulting diffuse radiation, D_t, received by an inclined plane can be expressed as:

$$D_t = D_h \{ 0.5 [1 + \cos\beta] [1 - F1 - F2] + F1 [a'/c'] + F2 [b'/d'] \}$$

where

D_t, D_h	=	diffuse radiation on incline, horizontal
β	=	tilt of plane from horizontal
F1, F2	=	diffuse radiation brightness coefficients
a′, b′	=	solid angles occupied by circumsolar zone and horizon band weighted by their respective incidence on the slope
c′, d′	=	the equivalent of a′ and b′ for the horizontal

The new simplified version of the model assumes that all the energy of the horizon band is contained in an infinitesimally thin region at 0°. The above equation then becomes:

$$D_t = D_h \{ 0.5[1 + \cos\beta] [1 - F1] + F1 [a'/c'] + F2 \sin\beta \}$$

The parametric representation of insolation conditions
This section of the model is empirical and establishes the value of the brightness coefficients, F1 and F2, as functions of the insolation conditions. The magnitude of these parameters are treated as functions of the following parameters:

i. the solar zenith angle, z

ii. the horizontal diffuse radiation (normalised to $d = D_h m / I_o$)

iii. the relative importance of direct radiation at the earth's surface,

expressed in the parameter = $[D_h + I] / D_h$.

where

m	=	relative air mass ($m = 1 / \cos z$)
z	=	solar zenith angle
I	=	direct radiation normal to the sun's rays
I_o	=	normal extraterrestrial radiation ($I_o = 1367\ W/m^2$)

The parameters F1 and F2 are expressed as:

$$F1 = F11 + d\,F12 + z\,F13$$
$$F2 = F21 + d\,F22 + z\,F23$$

The values of the parameters F11....F23 are presented in Table 1. These figures are for a 25° circumsolar half angle and are obtained from experimental data. Tests using these figures for a range of sites indicate that they are not site dependent (Perez, 1987).

Calculation method for Perez model

In summary, the calculation procedure follows the steps outlined below (note that in this example all angles are in radians and all insolation measurements must be in $kJ/hr/m^2$):

1. Input Data:

G_h	=	global radiation on the horizontal
D_h	=	diffuse radiation on the horizontal
ω	=	hour angle
	=	(solar time - 12.00) * 15 * π / 180
ϕ	=	latitude (positive for southern hemisphere)
α	=	circumsolar half angle (α = 25° = 0.436 rad)
n	=	Julian day, Jan 1st = 1 .... Dec 31st = 365
β	=	tilt of plane from horizontal
A_t	=	Plane azimuth (A_t = 0° for a north facing plane)

2. Calculate sky parameters:

δ	=	declination angle
	=	$-0.4093 \sin\{ 360/365 * (n + 284) * \pi / 180 \}$
E	=	solar elevation
	=	$\arcsin (\sin \delta \sin \phi + \cos \delta \cos \phi \cos \omega)$
z	=	solar zenith
	=	$\pi/2 - E$
A	=	solar azimuth
	=	$\arccos \{ (\sin E \sin \phi - \sin \delta) / (\cos E \cos \phi) \}$
θ_t	=	incidence angle of sun's rays on tilted plane
	=	$\arccos (\sin \beta \cos E \cos (A - A_t) + \cos \beta \cos E)$

3. Calculate model parameters:

I	=	$(G_h - D_h) / \sin(E)$
d	=	$D_h / [\cos(z) * I_o]$, $I_o = 4921.2$ kJ/hr/m^2
ϵ	=	$(D_h + I) / D_h$

4. Calculate Xh

If $z < \pi / 2 - \alpha$ then

ϕh	=	1
Xh	=	$\cos(z)$

else

ϕh	=	$(\pi / 2 - z - \alpha) / (2 * \alpha)$
Xh	=	$\phi h * \sin(\phi h * \alpha)$

5. Calculate Xt

If $\theta_t \leq \pi / 2 - \alpha$ then

Xt	=	$\phi h * \cos(\theta_t)$

else

if $\theta_t \leq \pi / 2 + \alpha$ then

ϕt	=	$(\pi / 2 - \theta_t + \alpha) / (2 * \alpha)$
Xt	=	$\phi h * \phi t * \sin(\phi t * \alpha)$

else

Xt	=	0

6. Calculate a and c

$$a' = 2 * (1 - \cos\alpha) * Xt$$
$$c' = 2 * (1 - \cos\alpha) * Xh$$

7. Calculate F1, F2

Look up F11...F23 in table

$$F1 = F11 + d * F12 + z * F13$$
$$F2 = F21 + d * F22 + z * F23$$

Table 1: Values for F11...F23

range of ε		25° circumsolar region					
from	to	F11	F12	F13	F21	F22	F23
1.000	1.056	-0.011	0.748	-0.080	-0.048	0.073	-0.024
1.056	1.253	-0.038	1.115	-0.109	-0.023	0.106	-0.037
1.253	1.586	0.166	0.909	-0.179	0.062	-0.021	-0.050
1.586	2.134	0.419	0.646	-0.262	0.140	-0.167	-0.042
2.134	3.230	0.710	0.025	-0.290	0.243	-0.511	-0.004
3.230	5.980	0.857	-0.370	-0.279	0.267	-0.792	0.076
5.980	10.080	0.734	-0.073	-0.228	0.231	-1.180	0.199
10.080		0.421	-0.661	0.097	0.119	-2.125	0.446

8. Calculate diffuse component

$$D_t = D_h * \{ 0.5 [1 + \cos\beta] * [1 - F1] + F1 * [a'/c'] + F2 * \sin\beta \}$$

This calculation procedure can be repeated for each hour of sunlight during the day and the results summed to obtain a figure representing the diffuse radiation received on a tilted surface during one day.

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